EFFECTS OF COMPRESSIBILITY AND NONISOTHERMAL CONDITIONS ON THE PERFORMANCE OF FILM COOLING

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The equations for a turbulent boundary layer are transformed in order to analyze the effects of compressibility and nonisothermal conditions on the performance of film cooling.

One way of examining the effects of compressibility and nonisothermal conditions near a wall is to transform the equations for the behavior of a fast compressible boundary layer to corresponding equations for a low-speed boundary layer, and, in particular, to equations for a liquid with constant physical properties [1-3]. These transformations are called respectively the l transformation and the i transformation; they are used below for the case of a fast compressible turbulent boundary layer on a planar adiabatic plate beyond a region of injection of gaseous coolant (film cooling) [4].

A bar above a quantity refers to the low-speed flow; then the symbols of [3] are used to give the following form to the transformation scheme in the general case. The equations

$$\rho u = \frac{\partial \psi}{\partial y}, \quad \rho v = -\frac{\partial \psi}{\partial x}, \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial \tau}{\partial y}, \tag{2}$$

$$\rho u \frac{\partial h^0}{\partial x} + \rho v \frac{\partial h^0}{\partial y} = \frac{\partial}{\partial y}(q + u\tau), \tag{3}$$

$$\rho u \frac{\partial m_i}{\partial x} + \rho v \frac{\partial m_i}{\partial y} = \frac{\partial \dot{m}_i}{\partial y}, \tag{4}$$

describe the behavior of the fast layer and become as follows with suitable choice of the scale functions for the transformation

$$\sigma \equiv \frac{\bar{\psi}(\bar{x}, \bar{y})}{\psi(x, y)}, \quad \eta \equiv \frac{\bar{\rho}}{\rho} \cdot \frac{\partial \bar{y}}{\partial y}, \quad \xi \equiv \frac{\partial \bar{x}}{\partial x}, \quad H \equiv \frac{\bar{h}}{h^0} \text{ and } M \equiv \frac{\bar{m}_i}{m_i} \tag{5}$$

and when all the quantities appearing in the equation are transformed to the corresponding quantities for the low-speed layer,

$$\bar{\rho}\bar{u} = \frac{\partial \bar{\psi}}{\partial \bar{y}}, \quad \bar{\rho}\bar{v} = -\frac{\partial \bar{\psi}}{\partial \bar{x}}, \tag{1a}$$

$$\bar{\rho}\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{\rho}\bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{d\bar{p}}{d\bar{x}} + \frac{\partial \bar{\tau}}{\partial \bar{y}}, \tag{2a}$$

$$\bar{\rho}\bar{u} \frac{\partial \bar{h}}{\partial \bar{x}} + \bar{\rho}\bar{v} \frac{\partial \bar{h}}{\partial \bar{y}} = \frac{\partial \bar{q}}{\partial \bar{y}}, \tag{3a}$$

$$\bar{\rho}\bar{u} \frac{\partial \bar{m}_i}{\partial \bar{x}} + \bar{\rho}\bar{v} \frac{\partial \bar{m}_i}{\partial \bar{y}} = \frac{\partial \bar{\dot{m}}_i}{\partial \bar{y}}. \tag{4a}$$

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It has been assumed in writing these equations that all the diffusion coefficients are equal and that the Lewis number is equal to 1.

It has been shown [2] that dependence of the first three functions in (5) only on the x coordinate is a sufficient condition for the convective terms in (2) to describe the behavior of a boundary layer on a planar plate after substitution of the corresponding quantities for the low-speed layer, and the same applies to the convective terms in (2a). It can be shown that the scale functions H and M should be constants if analogous requirements are to be met by the convective terms in (3), (3a), (4), and (4a); in fact, from (1) and (1a) we have that

$$\frac{\sigma}{\eta}=\frac{\bar{u}}{u}=\frac{\bar{u}_e}{u_e}, \tag{6}$$

$$\rho u=\frac{1}{\sigma}\,\bar{\rho}\bar{u}\,\frac{\partial\bar{y}}{\partial y}\ \text{and}\ \rho v=\frac{1}{\sigma}\left(\bar{\rho}\bar{v}\xi-\bar{\rho}\bar{u}\,\frac{\partial\bar{y}}{\partial x}\right)+\psi\,\frac{d\ln\sigma}{dx}\,. \tag{7}$$

We substitute (7) in the left part of (3) and use the expression for H to get

$$\rho u\,\frac{\partial h^0}{\partial x}+\rho v\,\frac{\partial h^0}{\partial y}=\left(\bar{\rho}\bar{u}\,\frac{\partial\bar{h}}{\partial\bar{x}}+\bar{\rho}\bar{v}\,\frac{\partial\bar{h}}{\partial\bar{y}}\right)\frac{\rho}{\bar{\rho}}\cdot\frac{\xi\eta}{\sigma H}+\bar{u}\bar{h}\,\frac{\bar{\rho}}{\sigma}\,J(H^{-1},\,\bar{y})$$

$$+\bar{v}\bar{h}\,\frac{\bar{\rho}}{\sigma}\,\xi\,\frac{\partial H^{-1}}{\partial y}+\bar{\psi}\,\frac{\partial\bar{h}}{\partial\bar{y}}\cdot\frac{\rho}{\bar{\rho}}\cdot\frac{\eta}{\sigma H}\cdot\frac{d\ln\sigma}{dx}+\bar{\psi}\bar{h}\,\frac{1}{\sigma}\cdot\frac{\partial H^{-1}}{\partial y}\cdot\frac{d\ln\sigma}{dx}, \tag{8}$$

where $J(H^{-1},\bar{y})\equiv\partial(H^{-1},\bar{y})/\partial(x,y)$ is a Jacobian. If the distance from the plate is much greater than the thickness of the boundary layer $|y,\bar{y}\gg\delta,\bar{\delta}|$, the left part of (8) and the first term on the right become zero, in accordance with (3) and (3a). The third and fourth terms on the right become zero by virtue of the behavior of $\bar{v}$ and $\bar{\psi}(\partial\bar{h}/\partial\bar{y})$, while the second and last terms are different from zero, with the last tending to infinity on account of y. The simplest condition that provides zero for the last term is $\partial H^{-1}/\partial y=0$, while for the second it is H = const, which has been assumed [3]. Analogous arguments may be applied also to M = const.

As $\sigma(x)$, $\eta(x)$, $\xi(x)$, H = const and M = const, we may express all the quantities for the high-speed case via the scale functions and the corresponding quantities for the low-speed one, eliminating quantities τ, q, and m_i, since there are no mathematical formulas for the variations in these across the layer when the boundary layer is turbulent. The law of transformation of the pressure follows from the equation

$$\frac{d\bar{p}}{d\bar{x}}=\frac{\sigma^2}{\xi\eta^2}\cdot\frac{\bar{\rho}_e}{\rho_e}\left[\frac{dp}{dx}+\rho_e u_e^2\,\frac{d}{dx}\ln\left(\frac{\eta}{\sigma}\right)\right], \tag{9}$$

while we get the following equations [3] for the displacement thickness, and also for the thicknesses for loss of momentum, energy, and mass of the component i:

$$\bar{\delta}^*=\frac{\eta\rho_e}{\bar{\rho}_e}\left[\delta^*-\int_0^{\delta}\left(1-\frac{\bar{\rho}_e\rho}{\bar{\rho}\rho_e}\right)dy\right],$$

$$\bar{\theta}/\theta=\bar{\varphi}/\varphi=\bar{\Omega}_i/\Omega_i=\eta\rho_e/\bar{\rho}_e,\ \bar{h}/\bar{h}_e=h^0/h_e^0\ \text{и}\ \bar{m}_i/\bar{m}_{i_w}=m_i/m_{i_w}\,. \tag{10}$$

We make the above substitution into (1)-(4) and compare the results with (1a)-(4a), which enables us to put the condition for transformation of the first to the second in the form

$$\bar{\tau}=\frac{\sigma^2}{\xi\eta}\left[\tau+\frac{d\ln\sigma}{dx}\int_y^{\delta}\psi\,\frac{\partial u}{\partial y}\,dy+\rho_e u_e^2\,\frac{d\ln(\eta/\sigma)}{dx}\int_y^{\delta}\frac{\rho}{\rho_e}\left(\frac{u^2}{u_e^2}-\frac{\bar{\rho}_e}{\bar{\rho}}\right)dy+\frac{dp}{dx}\int_y^{\delta}\left(1-\frac{\bar{\rho}_e\rho}{\bar{\rho}\rho_e}\right)dy\right], \tag{11}$$

$$\bar{q}=\frac{\sigma H}{\xi}\left[q+u\tau+\frac{d\ln\sigma}{dx}\int_y^{\delta}\psi\,\frac{\partial h^0}{\partial y}\,dy\right], \tag{12}$$

$$\bar{\dot{m}}_i=\frac{\sigma M}{\xi}\left[\dot{m}_i+\frac{d\ln\sigma}{dx}\int_y^{\delta}\psi\,\frac{\partial m_i}{\partial y}\,dy\right]. \tag{13}$$

The scale functions of (5) should be chosen so as to meet the conditions of (11)-(13), which are undoubtedly satisfied at the edge of the boundary layer because of the law chosen for transformation of the pressure and of the choice of the scale functions. Equations (11)-(13) may be put as follows as regards the conditions at the wall:

$$\bar{\tau}_w = \frac{\sigma^2}{\xi\eta}\left\{\tau_w + (\rho_e u_e \theta + \psi_w)\, u_e \frac{d\ln\sigma}{dx} - \rho_e u_e^2\left[\delta^* + \theta - \int_0^\delta \left(1 - \frac{\bar{\rho}_e \rho}{\rho \bar{\rho}_e}\right) dy\right] \frac{d\ln(\eta/\sigma)}{dx} + \frac{dp}{dx}\int_0^\delta \left(1 - \frac{\bar{\rho}_e \rho}{\bar{\rho}\rho_e}\right) dy\right\}, \quad (14)$$

$$\bar{q}_w = \frac{\sigma H}{\xi}\left[q_w + (\rho_e u_e \varphi + \psi_w)(h_e^0 - h_w)\frac{d\ln\sigma}{dx}\right], \quad (15)$$

$$\bar{\dot{m}}_{i_w} = \frac{\sigma M}{\xi}\left[\dot{m}_{i_w} + (\rho_e u_e \Omega_i + \psi_w)(m_{i_e} - m_{i_w})\frac{d\ln\sigma}{dx}\right] \quad (16)$$

and may be supplemented by the equations

$$\bar{\tau}_w = \frac{\bar{\rho}_w \bar{\mu}_w}{\rho_w \mu_w}\cdot\frac{\sigma}{\eta^2}\tau_w, \quad \bar{q}_w = \frac{\bar{\rho}_w \bar{\mu}_w \mathrm{Pr}_w}{\rho_w \mu_w \overline{\mathrm{Pr}}_w}\cdot\frac{H}{\eta} q_w$$

and (17)

$$\bar{\dot{m}}_{i_w} = \frac{\bar{\rho}_w \bar{\mu}_w \mathrm{Sc}_w}{\rho_w \mu_w \overline{\mathrm{Sc}}_w}\cdot\frac{M}{\eta}\dot{m}_{i_w},$$

which are derived on the assumption that laminar viscosity, laminar heat transfer, and laminar mass transfer occur at the wall. Here we may note that obedience to (14)-(17) is equivalent to transformation of the integral equations for the boundary layer [3] even in those cases where $\psi_w \neq 0$, $h_e^0 \neq$ const and $m_{i_e} \neq$ const.

However, obedience to (14)-(17) does not guarantee that the transform will provide the actual existing variation in the tangential stress $\bar{\tau}$, the density of the heat flux $\bar{q}$, and the flux density $\bar{\dot{m}}_i$ for the mass of component i across the boundary layer. If we assume that we have in the turbulent part of the boundary layer a mechanism for mass and energy transport such as to meet the generalized analogy, then for both fluxes we have

$$\frac{q}{\tau} = \frac{1}{\mathrm{Pr}_t}\cdot\frac{(\partial h/\partial y)}{(\partial u/\partial y)} = \frac{1}{\mathrm{Pr}_t}\left[\frac{(\partial h^0/\partial y)}{(\partial u/\partial y)} - u\right], \quad \frac{\bar{q}}{\bar{\tau}} = \frac{1}{\overline{\mathrm{Pr}}_t}\cdot\frac{(\partial\bar{h}/\partial\bar{y})}{(\partial\bar{u}/\partial\bar{y})} \quad (18)$$

and

$$\frac{\dot{m}_i}{\tau} = \frac{1}{\mathrm{Sc}_t}\cdot\frac{(\partial m_i/\partial y)}{(\partial u/\partial y)}, \quad \frac{\bar{\dot{m}}_i}{\bar{\tau}} = \frac{1}{\overline{\mathrm{Sc}}_t}\cdot\frac{(\partial\bar{m}_i/\partial\bar{y})}{(\partial\bar{u}/\partial\bar{y})}. \quad (19)$$

Here (18) and (19) define the effective Prandtl number Pr_t and Schmidt number Sc_t, with $\mathrm{Pr}_t \to \mathrm{Pr}_w$, and $\mathrm{Sc}_t \to \mathrm{Sc}_w$ near the wall; we transform the second equation in (18) via (5) and compare with (11) and (12) to get via the first equation in (18) that

$$\frac{1}{\overline{\mathrm{Pr}}_t}\left[\tau + \frac{d\ln\sigma}{dx}\int_y^\delta \psi\frac{\partial u}{\partial y}dy + \rho_e u_e^2\frac{d\ln(\eta/\sigma)}{dx}\int_y^\delta \frac{\rho}{\rho_e}\left(\frac{u^2}{u_e^2} - \frac{\bar{\rho}_e}{\bar{\rho}}\right)dy\right.$$

$$\left.+ \frac{dp}{dx}\int_y^\delta\left(1 - \frac{\bar{\rho}_e\rho}{\bar{\rho}\rho_e}\right)dy\right] = \left[\frac{1}{\mathrm{Pr}_t} - \left(\frac{1}{\mathrm{Pr}_t} - 1\right)u\frac{(\partial u/\partial y)}{(\partial h^0/\partial y)}\right]\tau + \frac{d\ln\sigma}{dx}\cdot\frac{(\partial u/\partial y)}{(\partial h^0/\partial y)}\int_y^\delta \psi\frac{\partial h^0}{\partial y}dy. \quad (20)$$

Similar operations with (11), (13), and (19) give

$$\frac{1}{\overline{\mathrm{Sc}}_t}\left[\tau + \frac{d\ln\sigma}{dx}\int_y^\delta \psi\frac{\partial u}{\partial y}dy + \rho_e u_e^2\frac{d\ln(\eta/\sigma)}{dx}\int_y^\delta \frac{\rho}{\rho_e}\left(\frac{u^2}{u_e^2} - \frac{\bar{\rho}_e}{\bar{\rho}}\right)dy\right.$$

$$\left.+ \frac{dp}{dx}\int_y^\delta\left(1 - \frac{\bar{\rho}_e\rho}{\bar{\rho}\rho_e}\right)dy\right] = \frac{1}{\mathrm{Sc}_t}\tau + \frac{d\ln\sigma}{dx}\cdot\frac{(\partial u/\partial y)}{(\partial m_i/\partial y)}\int_y^\delta \psi\frac{\partial m_i}{\partial y}dy. \quad (21)$$

Equations (20) and (21) are undoubtedly met near the wall, while system (14)-(17), (20), (21) enables one to find the scale functions σ, η and ξ in this case.

Consider the flow of a fast compressible gas near an impermeable adiabatic planner plate ($\psi_w = m_{iw} = q_w = 0$); if the scale functions nowhere become zero, we have from (17) that the corresponding low-speed flow will also pass along an impermeable adiabatic wall ($\bar{\psi}_w = \bar{m}_{iw} = \bar{q}_w = 0$). The injected gas differs in nature or temperature from that in the main flow, so in at least one of (15) and (16) the light part is not identically zero, and consequently, these equations can be met only for σ = const.* Since (20) and (21) are obeyed by the gradient free flow ($dp/dx = 0$) if η/σ = const and $Pr_t = \overline{Pr}_t = 1$, then flow at a low speed will correspondingly also have no gradient ($d\bar{p}/d\bar{x} = 0$), and analogous results are obtained if at the wall we have strong injections such that $\psi_w > 0$, $(\partial m_i/\partial y)_w = (\partial h/\partial y)_w = 0$.

This shows that the low-speed and high-speed flows have similar distributions in the current functions, projections of the velocity on the x axis, total enthalpy, and concentration. The last means, in particular, that the dimensionless enthalpy at the wall is the same in corresponding sections, i.e.,

$$\eta_{ef}(x) = \bar{\eta}_{ef}(\bar{x}), \tag{22}$$

where

$$\eta_{ef}(x) \equiv \frac{h_{wa}(\infty) - h_{wa}(x)}{h_{wa}(\infty) - h_{wa}(x_0)} \text{ and } \bar{\eta}_{ef}(\bar{x}) \equiv \frac{\bar{h}_e(\infty) - \bar{h}_{wa}(\bar{x})}{\bar{h}_e(\infty) - \bar{h}_{wa}(\bar{x}_0)}$$

represent the effectiveness of film cooling of the wall [4, 6], while x_0 and $\bar{x}_0$ are respectively the initial sections in which the profiles of these quantities are similar.

The edges of the slit for porous strip in front of the adiabatic wall (Fig. 1) define the similar sections and one can always state a current line passing through these edges, so σ should be defined by the ratio of the mass flow rates of the injected gas in the low-speed and high-speed cases. Then we can find the values of the scale functions of (5) on the transformation for an adiabatic plate in correspondence with (6), (14), and (17) as follows:

$$\sigma = \frac{\bar{M}_2}{M_2}, \quad \eta = \sigma \frac{u_e}{\bar{u}_e}, \quad \xi = \sigma\eta \frac{\rho_w \mu_w}{\bar{\rho}_w \bar{\mu}_w},$$
$$H = \frac{\bar{h}_e(\infty) - \bar{h}_{wa}(\bar{x}_0)}{h_{wa}(\infty) - h_{wa}(x_0)}, \quad M = \frac{\bar{m}_{ie}(\infty) - \bar{m}_{iw}(\bar{x}_0)}{m_{ie}(\infty) - m_{iw}(x_0)}, \tag{5a}$$

if $dp/dx = 0$ and $Pr_t = Sc_t = \overline{Pr}_t = \overline{Sc}_t = 1$. Here all the scale functions, apart from ξ, are constants, while the relation between the coordinates at the corresponding points is found from (5a) as

$$\frac{d\bar{x}}{dx} = \sigma\eta \frac{\rho_w(x)\mu_w(x)}{\bar{\rho}_w(\bar{x})\bar{\mu}_w(\bar{x})}$$

and

$$\frac{\partial \bar{y}}{\partial y} = \frac{\rho u_e}{\bar{\rho}\bar{u}_e} \cdot \frac{\bar{M}_2}{M_2}. \tag{23}$$

We write the second equation in (7) via (5a) for the conditions at the wall:

$$v_w = \bar{v}_w \frac{\eta}{\sigma} \cdot \frac{\sigma\mu_w}{\bar{\mu}_w} \frac{\bar{\rho}_w}{\rho_w} \cdot \frac{u_w}{\sigma} \left(\frac{\partial \bar{y}}{\partial x}\right)_w. \tag{7a}$$

This equation is always met for an impermeable wall $\psi_w = u_w = v_w = 0$; if the wall has strong injection at a constant temperature ($\psi_w \neq 0$, h_w^0 = const), then the derivative on the right in (7a) is zero, while the equation becomes

$$\frac{(v_w/u_w)}{(\bar{v}_w/\bar{u}_w)} = \frac{(v_w/u_e)}{(\bar{v}_w/\bar{u}_e)} = \frac{\sigma\mu_w}{\bar{\mu}_w}. \tag{7b}$$

This shows that the condition $\sigma\mu_w/\bar{\mu}_w = 1$ is compatible with the condition for similarity in the velocity distributions (u and v) near the wall for strong injection. In what follows we assume that

$$\frac{\sigma\mu_2}{\bar{\mu}_2} = \frac{\overline{Re}_2}{Re_2} = 1 \text{ and } \frac{(v_2/u_2)}{(\bar{v}_2/\bar{u}_2)} = \frac{(v_2/u_e)}{(\bar{v}_2/\bar{u}_e)} = 1. \tag{7c}$$

*In [2] the discussion concerned the case σ = var for m_i = const and h^0 = const.

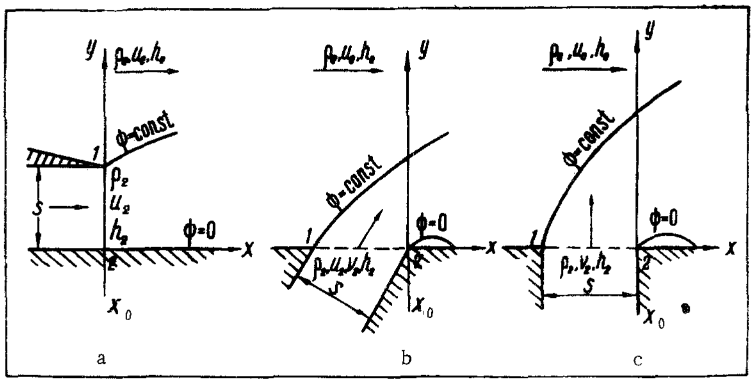

Fig. 1. Scheme of secondary gas supply to the main stream: a) tangential injection; b) injection at an angle; c) injection normal to the surface.

We integrate the first equation in (23) to relate the corresponding sections in the low-speed case ($\bar{x}$ = const) and the high-speed case (x = const), at which we have equality of the effective film cooling rates as defined by (22). Equations (5a) and (7c) in principle solve the problem for the injection of a gaseous coolant if these equations are supplemented with ones defining the physical properties of the gas mixture, for example, those of [5];

$$\rho\mu = \frac{1}{\sum_i \rho_i^{-1} m_i} \left(\sum_i \mu_i M_i^{-0,5} m_i \Big/ \sum_i M_i^{-0,5} m_i \right), \tag{24}$$

and

$$\mu_i = A_i T^{1,5}/(T + B_i) \quad \text{(Sutherland's formula)} \tag{25}$$

$$\rho_i = \rho_i(p,\ T) \quad \text{(equation of state).} \tag{26}$$

As an example we consider an air flow with injected cooling air. Here we need transform only the equations for continuity, motion, and energy, while in place of (24) we use the following equation [5]:

$$\frac{\rho\mu}{\rho_w \mu_w} = \left(\frac{p}{p_w}\right)^{0,992} \left(\frac{h}{h_w}\right)^{-0,343} \approx \left(\frac{h}{h_w}\right)^{-0,343}. \tag{27}$$

If the low-speed flow is also of air (l transformation) then we use (5a), (7c), (23), and (27) to get

$$\frac{\bar{x} - \bar{x}_0}{\bar{s}} = (h_2/h_2^0)^{0.343} \cdot \frac{x - x_0}{s} = \left(1 - \frac{k-1}{k+1}\lambda_2^2\right)^{0,343} \cdot \frac{x - x_0}{s}, \tag{28}$$

which implies that the performance in the high-speed flow will be higher than that in the low-speed one for the same dimensionless distance from the injection point. Equation (28) demonstrates the effect of the compressibility on the relation between the related sections in the two flows.

If the low-speed flow is that of a liquid with constant physical properties (i transformation), then we use (5a), (7c), (23), and (27) to get

$$\frac{\bar{x} - \bar{x}_0}{\bar{s}} = \int\limits_{(x_0/s)}^{(x/s)} \frac{\rho_w \mu_w}{\rho_2 \mu_2}\, d\left(\frac{x}{s}\right). \tag{29}$$

On the other hand, from (10), (23), (25), and (27) and the approximating equation for the performance of the film cooling [6]

$$\bar{\eta}_{\mathrm{ef},\,n}(\bar{x}) = C_n \left(\frac{\bar{x} - \bar{x}_0}{\bar{s}}\right)^{-n}, \tag{30}$$

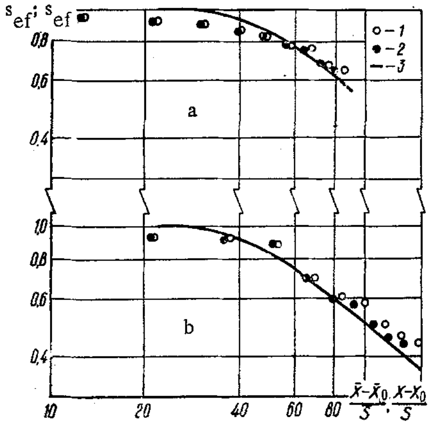

Fig. 2. Comparison of experimental and calculated values of the effectiveness of film cooling for a flat plate behind a tangential slot: a) experiment No. 11: $u_2 = 141$ m/sec, $u_e = 140$ m/sec, $T_2^0 = 345°K$, $T_e^0 = 967°K$, $s_{ef} = 5.5$ mm; b) experiment No. 5: $u_2 = 140$ m/sec, $u_e = 134$ m/sec, $T_2^0 = 320°K$, $T_e^0 = 952°K$, $s_{ef} = 3.2$; 1) experimental data [8]; 2) experimental data recalculated by means of (22), (29); 3) theoretical curves [9].

we get relationships for the converse:

$$\frac{x-x_{n=1}}{s} = \frac{\left(1+C_pB/h^0_{wa}(\infty)\right)\sqrt{(\bar{h}_2/\bar{h}_e)\left(1-\frac{k-1}{k+1}\lambda_2^2\right)}}{\left[C_pB/h^0_{wa}(\infty)+(\bar{h}_2/\bar{h}_e)\left(1-\frac{k-1}{k+1}\lambda_2^2\right)\right]} \times\left\{\frac{\sqrt{1-f(\bar{x})}}{f(\bar{x})}-\frac{\sqrt{1-f(\bar{x}_{n=1})}}{f(\bar{x}_{n=1})}+\frac{1}{2}\cdot\frac{C_pB-h^0_{wa}(\infty)}{C_pB+h^0_{wa}(\infty)} \times\ln\left[\frac{1+\sqrt{1-f(\bar{x})}}{1-\sqrt{1-f(\bar{x})}}\cdot\frac{1-\sqrt{1-f(\bar{x}_{n=1})}}{1+\sqrt{1-f(\bar{x}_{n=1})}}\right]\right\} f(\bar{x})\,\frac{\bar{x}-\bar{x}_0}{\bar{s}} \qquad (31)$$

and

$$\frac{x-x_n}{s} = \left[\frac{(\bar{h}_e/\bar{h}_2)}{1-\frac{k-1}{k+1}\lambda_2^2}\right]^{0,343}\left\{\sum_{m=0}^{N-1}\frac{C^m_{N-1}}{\left(1-\frac{m}{N+0,343}\right)} \times\left[\left(\frac{f(\bar{x})}{1-f(\bar{x})}\right)^{m-N-0,343}-\left(\frac{f(\bar{x}_n)}{1-f(\bar{x}_n)}\right)^{m-N-0,343}\right]\right\}[f(\bar{x})]^{N+0,343}\,\frac{\bar{x}-\bar{x}_0}{\bar{s}}, \qquad (31a)$$

where $f(\bar{x}) = [1-\bar{h}_{wa}(\bar{x}_0)/\bar{h}_e]\bar{\eta}_{ef}$ is a function of the coordinate $\bar{x}$, and $\bar{x}_n$ is the coordinate corresponding to the x_n at which (30) begins to apply; also, $N = 1/n-0.343$ is an integer greater than zero and C^m_{N-1} is a binomial coefficient. We wish to know the performance in film cooling with a liquid as a function of the distance from the injection point, which we approximate via parts of curves of the form of (30) with powers $n(n \le 1)$, and then we use (31) and (31a) to define the coordinates of the low-speed flow in relation to the coordinates of the high-speed one successively in accordance with the selected parts. Here $\bar{u}_e/u_e$ and H should be chosen, while σ and η should be obtained by calculation.

Equation (29) differs from (28) in incorporating not only the effects of compressibility but those also of nonisothermal conditions on the performance; the effects of the latter can be seen by putting λ_e and λ_2 equal to zero, when (29) shows that directly at the point of injection $\rho_w\mu_w/\rho_2\mu_2 \approx 1$, and the dimensionless coordinates along the abscissa axis are virtually the same for the low-speed and high-speed cases. Far from the point of injection we have $\rho_w\mu_w/\rho_2\mu_2 \approx \rho_e\mu_e/\rho_2\mu_2$ greater or less than unity in dependence on whether the injected gas is hotter or colder. In the first case, the performance decreases, while in the second case it increases relative to the performance of a flow of liquid with constant physical properties.

Petrov [7] has given experimental results on the performance of film cooling beyond a tangential slot on injecting nitrogen into an air flow along a planar plate when the velocity is supersonic ($\lambda_e = 1.48$). His experimental results were in almost complete agreement with the efficiency as calculated for low-speed flows and used in such a way that $\bar{u} = u$ and $\overline{Re}_2 \approx Re_2$. This agrees completely with our deductions, since, in this case, $\lambda_2 \le 0.5$, and from (28) we get $0.985 \le (\bar{x}-\bar{x}_0/\bar{s})/(x-x_0/s) \le 1$.

Borodachev [8] has given experimental results on the performance of film cooling behind a tangential slot when injecting air into an air flow ($u_2 \approx u_e$), where λ_e and λ_2 did not exceed 0.5, while the temperature change in the boundary layer was ± 310°. The ratio $\rho_w \mu_w / \rho_2 \mu_2$ was deduced from the temperature of the adiabatic walls, and the integral on the right in (29) was calculated to relate the coordinates along the abscissa axis for the low-speed and high-speed cases. Figure 2 shows the observed performance given in [8], and also the experimental results converted via (22) and (29), together with the theoretical performance from the data of [9], the theoretical calculation incorporating the assumption that dynamic boundary layers with injected material are closed at the exit from the slot, while the nomimal origin of the dynamic boundary layer at the wall is taken relative to the slot position such that the dimensionless current function $\xi_{wbl} = \bar{\psi}_{wbl}/\bar{\mu}$ is equal to $\xi_{wbl} = 0.5\,\overline{\mathrm{Re}}_2$, thus characterizing the flow in the dynamic boundary layer near the wall at the slot section. It is clear from the results that the nonisothermal boundary layer in this case has little effect on the performance of the film cooling, and the latter can be calculated quite accurately via the equations for the performance of film cooling with a quasiisothermal flow [10].

The equation transformation method of [1-3] can be used to take into account compressibility and nonisothermal conditions as regards the performance of film cooling. These effects can be neglected over a comparatively wide range in velocity and temperature if one retains certain definite conditions for comparing the high-speed and low-speed flows.

NOTATION

x, y	are rectangular coordinates of which the abscissa coincides with the surface of the flat plate and the direction of the undisturbed flow;
s	is a characteristic dimension (height of slot, length ot porous section);
ρ	is the density;
μ	is the viscosity;
ψ	is the flow function;
ξ_{wbl}	is the dimensionless flow function;
u, v	are the projections of the velocity vector on the x and y axes, respectively;
p	is the pressure;
T	is the temperature;
h	is the enthalpy;
C_p	is the specific heat at constant pressure;
k	is the isoentropic component
λ	is the velocity coefficient, the ratio of the flow velocity to the critical velocity;
m_i	is the mass flow density of the i-th component;
M_i	is the molecular weight of the i-th component;
M_2	is the mass flow rate of gas per unit width of flow;
τ	is the shear stress;
q	is the heat flux density;
σ, η, ξ, H, M	are scale functions;
$\delta,\ \delta^* = \int_0^\delta \left(1-\frac{\rho u}{\rho_e u_e}\right) dy,$	are respectively the thickness, thickness of displacement;
$\theta = \int_0^\delta \frac{\rho u}{\rho_e u_e}\left(1-\frac{u}{u_e}\right) dy,$	is the momentum thickness;
$\varphi = \int_0^\delta \frac{\rho u}{\rho_e u_e}\,\frac{h_e^0 - h^0}{h_e^0 - h_w}\, dy,$	is the energy thickness;
$\Omega_i = \int_0^\delta \frac{\rho u}{\rho_e u_e}\cdot\frac{m_{i_e} - m_i}{m_{i_e} - m_{i_w}}\, dy$	is the mass thickness of the i-th component of the boundary layer;
$\mathrm{Re}_2 = M_2/\mu_2$	is the Reynolds number;
Pr	is the Prandtl number;
Sc	is the Schmidt number;
A_i, B_i, C_n, N, n	are constants.

Subscripts

0 denotes total parameters;
e denotes undisturbed flow;

2 denotes injected flow;
w denotes wall;
wa denotes adiabatic wall;
i denotes the i-th component of the gas mixture;
ef denotes effective;
wbl denotes well boundary layer.

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